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LETTER TO THE EDITOR

Critical slowing down in a randomly biased percolating system at percolation threshold

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Abstract. Using computer simulation, a random walk motion is studied in a random percolating system at percolation threshold in the presence of a random field. The RMS displacement is found to decrease systematically as a function of field strength and shows a non-universal power law dependence on it. The long time behaviour seems similar to the critical slowing down recently predicted in an analogous magnetic system. The spectral dimensionality varies with the field strength and is found to be less than 1.

Although the thermodynamics of the Ising model and diffusion in percolating systems seem quite different, they share some common features: the manner in which the thermal fluctuations develop in an Ising model and the way the root mean square (RMS) displacements increase in percolating systems as the time increases seem to behave similarly. In fact, for the last few years it has been believed that the dynamic scaling (Hohenberg and Halperin 1977) exponent z for a homogeneous thermal system ($\tau \sim \xi_t^z$ with τ the relaxation and ξ_t the thermal correlation length) is analogous to the asymptotic power law exponent or walk dimension in anomalous diffusion (Alexander and Orbach 1982, Gefen *et al* 1983, Rammal and Toulouse 1983, Pandey *et al* 1984, Havlin and Ben-Avraham 1983, Stauffer 1985a, b) in percolating systems $R \sim t^k$ ($k = 1/z$) where R is the RMS displacement in time t. (The dynamical scaling has also been used to describe a variety of systems including kinetic growth and ungrowth in recent years (Kerstein 1986, Vicsek and Family 1984).) Let us attempt to trace out a possible one-to-one correspondence between the two phenomena. In the Ising model, thermal fluctuations are caused by flipping the spins (up or down), while in a percolating system, geometrical fluctuations are caused by adding or removing the sites (occupied or empty sites in a binary system). Both excitations are of local nature and are related, since in the dilute Ising model the percolation crossover exponent ϕ (the ratio of the percolation correlation length exponent and the thermal correlation length exponent) is unity (Stinchcombe 1983). It is therefore intuitive to believe that the thermal fluctuations propagate through the thermal clusters (of up or down spins) almost in the same fashion as a diffuser passes through the percolating cluster (the conductivity of a connected path depends upon the geometrical fluctuations). Now, if a random magnetic field is present at each site in the Ising model, then one may argue that it

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may affect the thermal fluctuations in the same way as a random electric field affects the diffuser in percolating systems.

Using scaling theory supported by renormalisation group arguments Fisher (1986) has recently predicted that in a random field Ising model, the relaxation time τ for the thermal fluctuations diverges exponentially with the thermal correlation length ξ_t as the thermal critical point is approached. Here we present results from our computer simulations for the random walk motion in percolating systems to see whether a similar critical slowing down occurs for the anomalous diffusion in percolating fractals in the presence of random fields. Our randomly biased diffusion is related to ordinary biased diffusion (Barma and Dhar 1983, Pandey 1984, Stauffer 1985a, b) in the same way as the Ising model in a random field is related to that in a homogeneous field, or as resistor-diode networks with random orientation of the diodes are related to networks with all diodes oriented in the same direction (Redner 1982).

We consider a simple cubic lattice. A fraction p of these sites chosen randomly are occupied and the rest of the sites of concentration $1-p$ are empty. Clusters (Stauffer 1985a,b) (formed by joining the nearest-neighbour sites) of various sizes appear. The motion of a particle is restricted only to the occupied sites. We will concentrate here on the critical concentration p_c where an infinite cluster (along with many smaller clusters of various sizes) appears for the first time on increasing the concentration of the occupied sites from p below p_c. (For the simple cubic lattice $p_c = 0.3117$.) Each of the occupied sites is then assigned one of the six directions chosen randomly. A particular value is set for the field intensity B which determines the probability that a particle once at a site will attempt to hop in its preset random direction. One of the occupied sites (called the local origin), say i, is chosen randomly and a particle (diffuser or 'ant') is placed on it. To decide in which direction it will attempt to jump, a pseudorandom number is selected and compared with B; if it is less than B, then the neighbouring site j in the preassigned direction from the site i is chosen. Otherwise any of the six neighbouring sites, say k, is chosen randomly. If the hopping site j or k (chosen according to the prescription) is occupied then the random walker is moved to it; otherwise the diffuser stays at its old site i. The time is increased by unity in either case. The process of selecting a prescribed neighbouring site, an attempt to move the particle to it and updating time and displacement is repeated again and again for a preset (maximum) number of steps. This whole procedure is repeated for several local origins and for many independent samples to obtain a reliable estimate for the average of the quantities such as RMS displacement, number of distinct visited sites, etc. A sample is prepared by occupying sites of concentration p_c and assigning random directions to them as described above. We have used periodic boundary conditions on a $50 \times 50 \times 50$ cubic lattice.

In the absence of a random field (i.e. $B = 0$), the random walk motion in a percolating system is well understood at $p = p_c$, where the infinite cluster is self-similar up to infinite length scale. Scaling arguments (Gefen *et al* 1983), verified by computer simulations (Pandey *et al* 1984, Havlin and Ben-Avraham 1983), suggest an anomalous power law behaviour for the RMS displacement $R \sim t^k$, with $2k = (2\nu - \beta)/(2\nu + \mu - \beta)$, where ν and β are the percolation correlation length and volume fraction exponents, respectively, and μ is the conductivity exponent. Thus in $d = 3$, $k \simeq 0.2$, as is also verified in our simulation here (see figure 1). When switching on a small random field (of strength $B = 0.1$) we observe a decrease in the magnitude of the RMS displacement, as well as in its asymptotic power law exponent k. As is evident from figure 1, the trend of slowing down motion continues systematically as the magnitude of the random

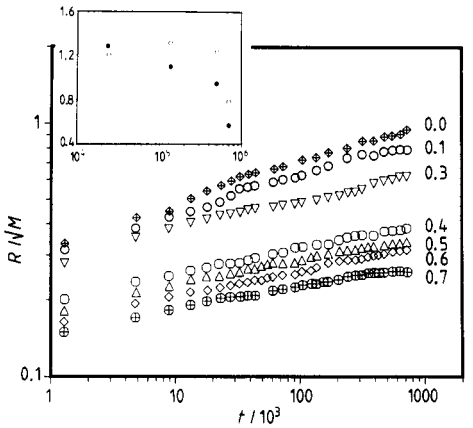

Figure 1. $R/\sqrt{M}$ against t plot on a log-log scale, R = RMS displacement, M = number of local origins, $(N)X$ = number of samples (run). Sample size $50\times50\times50$. The field intensities are $B = 0.0$, 0.1, 0.3, 0.4, 0.5, 0.6, and 0.7 for $N = 50$, run = 20. The insert shows a typical variation of the effective exponent $-10\,k$ (●) and k' (○) (see text) with time t on a semilog scale.

field strength B increases. Furthermore, if we fit these data with $R \sim t^k$, then we obtain different values of the exponent k for different values of B. For example, at $B = 0.4$, $k \simeq 0.12$, while at $B = 0.7$ a much smaller (varying) value of k is obtained (see the insert of figure 1). At higher values of B, the motion becomes too slow and perhaps the relaxation time becomes very large. Finally at the extreme value of $B(=1)$, k becomes zero. We have calculated the power law exponent k at different time intervals and, at $B = 0.7$, variation of the effective k with time t is shown in the insert of figure 1. The continuously decreasing magnitude of k with t seems to suggest that either we are not in the asymptotic regime, or the simple power law $R \sim t^k$ does not hold.

Based on the recent scaling arguments (Fisher 1986), if we assume a logarithmic variation of the RMS displacement, $R \sim (\ln t)^{k'}$, then we obtain a variation in the effective exponent k', as shown in the insert of figure 1 for $B = 0.7$. It appears that $k' \simeq 1.25 \pm 0.1$ for the time regime up to $t \sim 5\times10^5$ and thereafter there is a sharp drop in its value $k' \simeq 0.75 \pm 0.1$. Such a sharp change is also observed in the variation of the effective power law exponent k in the same regime. Thus, the simulation data suggest two types of relaxation with two exponents (for the faster relaxation is followed by the slower).

If we use the relation (Alexander and Orbach 1982, Rammal and Toulouse 1983) $S \sim t^{d_s/2}$ for the number of distinct visited sites S in time t, we may evaluate the spectral dimensionality d_s. A plot of S against t on a log-log scale is presented in figure 2. For the typical field intensities $B = 0.4$ and 0.7, the effective spectral dimensionality turns out to be about 0.57 ± 0.03 and 0.39 ± 0.03, respectively. On the other hand, the Alexander-Orbach conjecture (Alexander and Orbach 1982) suggests a superuniversal value $d_s \simeq \frac{4}{3}$ for the percolating systems at percolation threshold. The spectral dimensionality d_s for most of the self-similar systems (Alexander and Orbach 1982, Rammal and Toulouse 1983, Mandelbrot 1984) seems to lie between 1 and 2, while in a local symmetric breaking percolating system as we see here the spectral dimensionality is less than one and that, too, it depends on the field intensity. Very recently, in a different context, the systems, which have spectral dimensionality less than one, have been called (Kopelman *et al* 1986) 'fractal dust'. In our case, it is

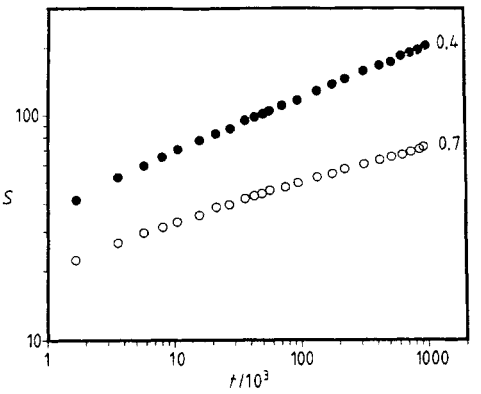

Figure 2. Number of distinct visited sites S against $t/10^3$ log-log plot for the field intensities given with the data points. Statistics are the same as in figure 1.

tempting to call such a system (a random field percolating system) 'fracton glass'. Here, the percolating fractal is known to be a homogeneous self-similar quenched random system (Leyvraz and Stanley 1983) and the inhomogeneity is caused by the random bias field. Note that in percolating systems below their percolation threshold the spectral dimensionality is less than one, but there is no global transport. Contrary to this is the percolating system at percolation threshold in random biased field where there is a global transport. The local fields along with the random geometry introduce local trapping barriers, which makes the transport too slow to distinguish from the metastability as in glasses (Edwards and Anderson 1975).

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